Example (of a Left-tailed test)

A newspaper claims that the percent of New Yorkers who have already contracted and recovered from covid-19 is less than 15%. A random sample of New Yorkers found outside their homes found that 13.9% of the 3,000 people selected for the sample tested positive. Use the sample information to test the newspaper's claim.

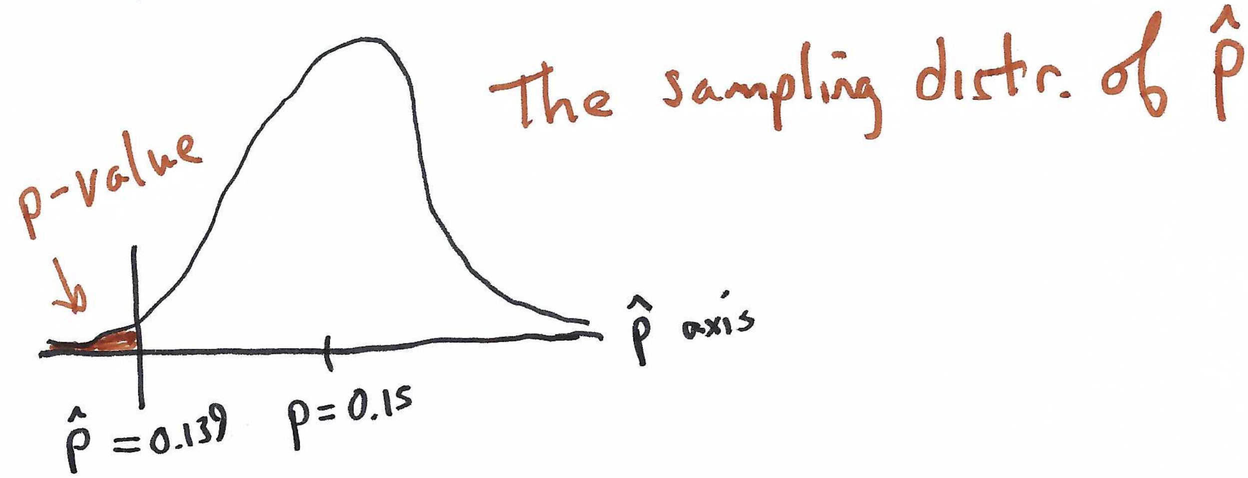
Soln

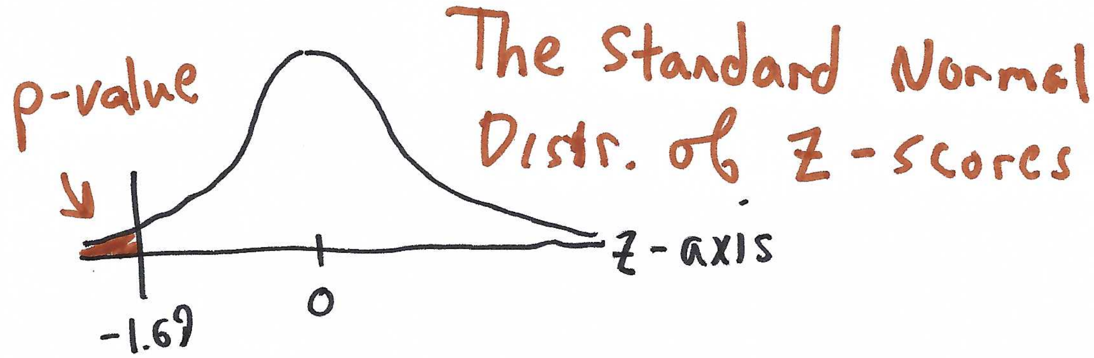
Step 1

Hypotheses

$H_0: p = 0.15$ ← (or $p \geq 0.15$)

$H_A: p < 0.15$ ← Newspaper's claim





Step 2 Conditions Check

① The problem states that the sample is random.

② $n \times p_0 = 3000(0.15) = 450$ and

$n \times (1-p_0) = 3000(1-0.15) = 2{,}550.$

(p_0 is the percent listed in H_0 and H_A)

So, there are at least 10 successes (people who test positive for Covid-19 antibodies) and at least 10 failures (people who test negative for the antibodies) expected in a sample of 3,000 New Yorkers.

So, we can assume that the sampling dist. of $\hat{p}$ is approximately normally distributed!

Step 3 Find the test statistic

The test statistic is the z-score of your sample statistic, $\hat{p} = 0.139$.

I get this number from doing a 1-prop-Z-test on the calculator

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.139 - 0.15}{\sqrt{\frac{0.15(1-0.15)}{n}}} \approx \boxed{-1.69}$$

standard deviation lengths.

So, $\hat{p} = 13.9\%$ is about 1.69 standard error lengths below the mean of the sampling dist., $\mu_{\hat{p}} = p_0 = 0.15$.

Step 4 Find the p-value and make a decision as to whether or not to reject H_0.

For a left-tailed test, the

I get this number from doing a 1-prop-Z-test on the calculator.

$$\text{P-val} = P(\hat{p} < 0.139, \text{ assuming } H_0 \text{ is correct})$$
$$= P(z < -1.69) = \boxed{0.0458} \text{ (or } 4.58\%)$$

Step 4 Make a decision to either "reject H_0" or "fail to reject H_0" based on this guideline:

If p-val $\leq \alpha$, reject H_0.

If p-val $> \alpha$, fail to reject H_0

(If no value for α, the level of significance of the test is stated in the word problem, we use $\alpha = 0.05$ or 5%.)

Since the p-value $< \alpha$, we reject H_0 and support the H_A.

It is sufficient to write only this here.

I just give more explanation here

The practical significance level (the p-value) is 4.58%, or 0.0458. So, in the long run only about 458 samples in 10,000 would result in a Z-value (and sample proportion) as or more extreme than the values obtained from the sample. So, there is an about 4.58% chance we are making a mistake by deciding to reject H_0 (a type I error).

Step 5

Conclusion

Since we rejected the hypothesis, there is convincing sample evidence to support the newspaper's claim.

Right-tailed test Example

10.11 x = the number of people in the sample who said they approve of casino gambling

$x = 1035$

n = sample size = 1523

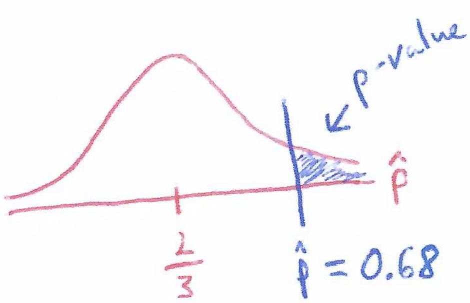
$\boxed{\alpha = 5\% = 0.05}$

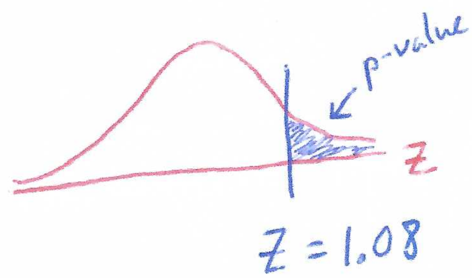
$\hat{p} = \frac{x}{n} = \frac{1035}{1523} \doteq 0.68$ = the % of people from the sample who approve of casino gambling

1 **Hypotheses**

$H_0: p = \frac{2}{3}$

$H_A: p > \frac{2}{3}$





2 **Conditions Check:**

① The problem states that the sample is a random sample.

② There are 1015 expected successes in the sample and 1523 − 1015 = 508 expected failures. So, there is at least 10 successes and 10 failures expected in a sample. We can assume the sampling distribution of $\hat{p}$ is approximately normal.

3 **Test Statistic**

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.68 - 0.667}{\sqrt{\frac{0.667(1-0.667)}{1523}}} \doteq \boxed{1.08}$$

(or 1.07 if you use the calculator 1-Prop-Z-te

4 **p-value**

The p-value $= P(\hat{p} > 0.68) = P(z > 1.08) = \text{ncdf}(1.08, 10^9, 0, 1) \doteq \boxed{0.1400}$

(or 0.1425 if you use 1-prop-Z-

Since the p-val > 0.05, **fail to reject H_0.**

5 **Conclusion**

There is **not convincing evidence** that more than 2/3 of American adults approve of casino gambling. The reason we observed a sample percent = to 68% is attributed to sampling variability.

10.72 | $n = 1010$ randomly selected American adults.

Left-tailed test

x = the number of successes $= 374$

$\hat{p} = \frac{x}{n} = \frac{374}{1010} = 0.37$

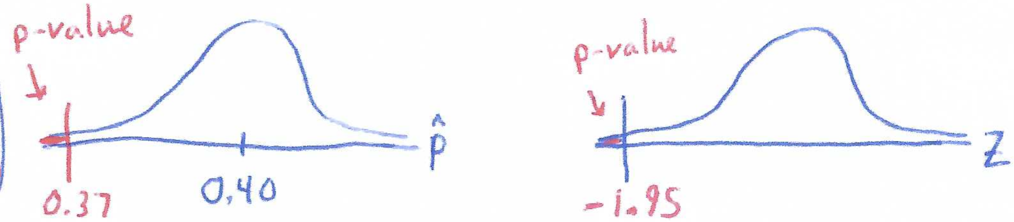
$\boxed{\alpha = 5\% = 0.05}$

p = the percentage of U.S. adults who believe the investment would result in a sum of over \$100,000.

Hypotheses

$H_0: p = 0.40$

$H_A: p < 0.40$



Conditions Check

1) The problem states that the sample is random

2) There are 404 successes and $1010 - 404 = 606$ failures expected in the sample.

Test Statistic

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}} = \frac{0.37 - 0.40}{\sqrt{\frac{0.40(1-0.40)}{1010}}} \doteq \boxed{-1.95}$$

or -1.93 if you use the calculator's 1-prop-Z-test

P-value

The p-value $= P(\hat{p} < 0.37) = P(z < -1.9) = \text{ncdf}(-10^9, -1.95, 0, 1)$

$= \boxed{0.0256}$

or 0.0270 if you use the calculator's 1-prop-Z-test

Since the p-value $\leq \alpha$, reject H_0

Conclusion

There is convincing sample evidence that the percentage of American adults who think the investment would result in a sum of over \$100,000 is less than 40%.